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# Synthesis of Generalized Laplace-Meijer (LK) Transforms and Meijer G-Functions: Extensions and Applications in Sustainable Transportation and Industrial Resilience

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**Abstract:** *This paper explores combinations of the generalized Laplace-Meijer (LK) transform, originally developed in distributional settings, with contemporary engineering challenges. Building on the representation theorem for the LK transform (Gulhane & Gudadhe, 2006), we synthesize its kernel involving modified Bessel functions expressed via Meijer G-functions with applications in battery sizing for plug-in hybrid electric vehicles, advancements in automotive batteries, and resilience modeling in global supply chains. Novel combinations include using LK/Meijer G transforms for solving differential equations governing battery dynamics, energy management in hybrid architectures, and stochastic modeling of supply chain disruptions. Practical examples demonstrate improved analytical solutions for impulsive signals and system responses in EV contexts. Results highlight enhanced modeling accuracy and resilience strategies, contributing to sustainable transportation and industrial systems.*

**Keywords:** Laplace-Meijer transform, Meijer G-function, generalized integral transforms, distributional functions, representation theorem, battery sizing, plug-in hybrid electric vehicles, automotive batteries, supply chain resilience, sustainable transportation, fractional transforms.

## I. NTRODUCTION

The Laplace-Meijer (or Meijer G-related) transform is a generalization of the classical Laplace transform that leverages the Meijer G-function (introduced by C.S. Meijer in the 1930s–40s). The Meijer G-function is an extremely versatile special function expressed as a Mellin-Barnes contour integral, encompassing many classical functions (hypergeometric, Bessel, Whittaker, etc.) as special

The generalized Laplace-Meijer (LK) transform provides a powerful framework for handling generalized functions and solving integral equations in distributional spaces. Gulhane and Gudadhe (2006) established a structure formula (representation theorem) for this transform in the dual space $LK'_{\mu}$, using kernels of the form $e^{-s}(xy)^{1/2}K_{\mu}(xy)$, directly linkable to Meijer G-functions.

This work combines that theoretical foundation with applied research in: battery sizing and hybrid vehicle architectures, advancements in automotive batteries, resilience in global and internal supply chains. Such combinations address gaps in modeling complex, non-smooth phenomena (e.g., impulsive loads in EVs or disruptions in supply chains) where classical transforms fall short.

The distributional Laplace transform is defined as $F(s) = \langle f(t), e^{-st}\rangle$for $\mu$either zero or a complex number with positive real part.

If $\mu$ is zero or a complex number with positive real part, the K transform $F(y)$is defined by $F(y) = \langle f(x), (xy)^{1/2}K_{\mu}(xy)\rangle \mathrm{Re}\ y > 0$where $K_{\mu}(z)$denotes the modified Bessel function of the third kind

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and order $\mu$. Similar to the K transformation, there is the I transformation $F(y) = \langle f(x), (xy)^{1/2} I_\mu(xy) \rangle$where $I_\mu(z)$denotes the modified Bessel function of the first kind and order $\mu > 1/2$.

We shall make considerable use of the differential operator $S_\mu = t^{-1/2} D t^{2\mu+1} D t^{\mu+1/2}$of second order i.e. $S_\mu = M_\mu N_\mu$, where $M_\mu$and $N_\mu$are defined in section 5.3 of Zemanian [1].

The notation and terminology follow Zemanian [1].

This paper deals with distributional generalized Laplace–Meijer (LK) transform of distributional function $f$in dual space of $LK_\mu$i.e. $LK'_\mu$which is defined by $LK_\mu(f(x)) = F(s, y) = \langle f(x), e^{-s}(xy)^{1/2} K_\mu(xy) \rangle$for complex parameters $s$and $y$.

The RHS has sense for $f \in LK'_\mu$and $e^{-s}(xy)^{1/2} K_\mu(xy) \in LK_\mu$.

The present paper mainly provides a representation theorem for the defined generalized transform.

## II. LITERATURE REVIEW

The combinations enhance analytical tractability over numerical methods alone, aligning theoretical math legacy with practical engineering impact. Limitations: Convergence conditions for specific parameters. Future: Fractional LK variants and AI-optimized kernels.

The combinations of the generalized Laplace-Meijer (LK) transform with applied engineering domains, as explored in this paper, demonstrate the enduring relevance of distributional mathematical tools in addressing real-world complexities. By leveraging the representation theorem established by Gulhane and Gudadhe (2006), which expresses distributions via weighted derivatives of bounded measurable functions using the kernel $e^{-s}(xy)^{1/2} K_\mu(xy)$(linkable to Meijer G-functions), we have bridged pure mathematical theory with practical challenges in battery sizing for plug-in hybrid electric vehicles, automotive battery advancements, and supply chain resilience modeling

## III. THE REPRESENTATION THEOREM

The generalized Laplace-Meijer (LK) transform provides a powerful framework for handling generalized functions and solving integral equations in distributional spaces. Gulhane and Gudadhe (2006) established a structure formula (representation theorem) for this transform in the dual space $LK'_\mu$, using kernels of the form $e^{-s}(xy)^{1/2} K_\mu(xy)$, directly linkable to Meijer G-functions.

This work combines theoretical foundation with applied research in battery sizing and hybrid vehicle architecture, advancements in automotive batteries, and resilience in global and internal supply chains. Such combinations address gaps in modeling complex, non-smooth phenomena (e.g., impulsive loads in EVs or disruptions in supply chains) where classical transforms fall short.

**Theorem** : Let $f(t, x)$be an arbitrary element of $LS'_\mu(\Omega)$(or $LK'_\mu(\Omega)$) and $\varphi(t, x)$be an element of $D(\Omega)$, the space of infinitely differentiable functions with compact support on $\Omega$. Then there exist bounded measurable functions $g_{\mu,r,v}(t, x)$(or $g_{\mu,r}(t, x)$) defined over $\Omega$such that

$$\langle f, \varphi \rangle = \langle \sum_{r=0}^{n} \sum_{v=0}^{m} (-1)^{r+v} e^{\mu t} j_\mu(t, x)\, D_t^r (D_t + S_t)^v\, g_{\mu,r}(t, x), \varphi(t, x) \rangle$$

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where $r$and $v$are appropriate non-negative integers satisfying $m \leq r+1$and $n \leq v+1$, and the sum is finite. Here:

$j_\mu(t,x)$is the weight function: $x^{\mu/2}$for Re $\mu > 0$, and $\left[x^{\mu/2}h(t,x)\right]^{-1}$for $\mu = 0$.
$S_\mu$is the second-order differential operator defined as
$S_\mu = t^{-1/2}D^{2\mu+1}t^{\mu+1/2}D$(or equivalently $S_\mu = M_\mu N_\mu$).
$D_t$ denotes differentiation with respect to $t$. This representation expresses any continuous linear functional $f$(distribution) as a finite sum of derivatives (in a weighted sense) of bounded measurable functions. It is the distributional analogue of the classical structure theorems.
Let $f(t,x)$be an arbitrary element of $LS'_\mu(\Omega)$(or $LK'_\mu(\Omega)$) and $\varphi(t,x)$an element of $D(\Omega)$, the space of infinitely differentiable functions with compact support on $\Omega$. Then there exist bounded measurable functions $g_{\mu,r,v}(t,x)$defined over $\Omega$ such that

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where $r$and $v$are appropriate non-negative integers satisfying $m \leq r+1$and $n \leq v+1$.
Here:
$j_\mu(t,x)$is the weight function: $x^{\mu/2}$for Re $\mu > 0$, and $\left[x^{\mu/2}h(t,x)\right]^{-1}$for $\mu = 0$.
The functions $g_{\mu,r,v}(t,x) \in L^\infty(\Omega)$.
This expresses any continuous linear functional $f$(a distribution) as a finite sum of weighted derivatives of bounded measurable functions — the distributional analogue of classical structure theorems.
Proof. Let $\{\rho'_{\lambda,\mu}\}$be the sequence of seminorms defining the topology on the testing space $LK_\mu$. Let $f \in LK'_\mu(\Omega)$and $\varphi \in D(\Omega)$. By the boundedness property of generalized functions (Zemanian [4], p. 70), there exists a constant $C > 0$and non-negative integers $r, v$(with $|\ l\ | \leq r, |\ q\ | \leq v$) such that

$$|\ \langle f,\varphi\rangle\ | \leq C \max_{|l|\leq r,|q|\leq v} \rho'_{\lambda,\mu}\varphi(t,x).$$

This is bounded by

$$|\ \langle f,\varphi\rangle\ | \leq C' \max_{m\leq r,n\leq v}\ \sup_{t,x}\ |\ e^{\mu t}j_\mu(t,x)\, D_t^l S_t^k \varphi(t,x)\ |$$

for suitable $m, n$. Change of Variable Define

$$\varphi_{n,v}(t,x) := e^{\mu t}j_\mu(t,x)\,\varphi(t,x), m \leq r, n \leq v.$$

Clearly, $\varphi_{n,v} \in D(\Omega)$since $\varphi$ has compact support and $j_\mu$is smooth where needed. Derivative Estimates (Induction on Orders) We must bound higher-order derivatives $|\ D_t^l S_t^k \varphi\ |$in terms of $\varphi_{n,v}$. Consider three cases:

Case 1: Re $\mu > 0$(where $j_\mu(t,x) = x^{\mu/2}$) Assume supp $\varphi =$ supp $\varphi_{n,v} = [A_1, B_1]$. Using the positivity of weighting terms like $e^{x^2}(1+x)^2x^{\mu/2} > 0$, Leibniz rule yields:

$$|\ D_t^l S_t^k \varphi\ | \leq C_1 e^{-x^2}x^{-(\mu/2)}\ |\ \varphi_{n,v}\ | + |\ D_t\varphi_{n,v}\ | + \cdots + |\ D_t^l D_t^k \varphi_{n,v}\ |$$

where $C_1$is the max coefficient from expansion.

 

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Case 2: $\mu = 0$and $0 < x < e^{-1}$Here $j_\mu(t,x) = [x^{\mu/2} h(t,x)]^{-1}$with $h(t,x) = \ln t$(adjusted per definition). Positivity of $| e^{-x^2} x^{1/2} \log x | > 0$gives analogous bound with logarithmic term.

Case 3: $\mu = 0$and $e^{-1} < x < \infty$Uses $| e^{-x^2} x^{1/2} | > 0$, yielding

$$| D_t^l S_t^k \varphi | \leq C_3 e^{-x^2} x^{1/2} | \varphi_{n,v} | + \text{lower-order terms.}$$

Inductive Step: By repeated application (induction on derivative orders), for obvious constant $C$:

$$| D_t^l S_t^k \varphi | \leq C e^{-x^2} [j_\mu(t,x)]^{l-1} \sum_{c \leq l, d \leq k} | D_t^c (D_t + S_t)^d \varphi_{n,v} |.$$

Substitute into Boundedness Inequality Plugging the inductive estimate into (1):

$$| \langle f, \varphi \rangle | \leq C'' \max_{n \geq n_0, v \geq v_0} \sup \sum_{c \leq m, d \leq n} | D_t^c (D_t + S_t)^d \varphi_{n,v} |.$$

Embedding into $L^1$Spaces. Note that

$$\sup \; | \varphi(t,x) | \leq \sup \; | \int \cdots \int D_t^l (D_t + S_t)^k \varphi(t,x) \, dt |$$

by fundamental theorem of calculus / integration by parts, using compact support. Thus, the functional is controlled by $L^1$norms of the higher derivatives of $\varphi_{n,v}$.

Let $(L^1)^2$denote the product space. Define the linear mapping $\tau: D(\Omega) \to (L^1)^2$by $\varphi \mapsto (D_t^m (D_t + S_t)^n \varphi_{n,v})$(one-to-one). The functional $\varphi \mapsto | \langle f, \varphi \rangle |$is continuous on $\tau D(\Omega)$with the $(L^1)^2$topology.

Step 5: Hahn-Banach Extension + Duality By Hahn-Banach theorem, extend to a continuous linear functional on all of $(L^1)^2$. Since the dual of $(L^1)^2$is isomorphic to $(L^\infty)^2$(Zemanian [4], pp. 214, 259), there exist $g_{m,n} \in L^\infty(\Omega)$(bounded measurable) such that

$$| \langle f, \varphi \rangle | \leq \sum \langle g_{m,n}, D_t^m (D_t + S_t)^n \varphi_{n,v} \rangle.$$

Recover the Representation Using the properties of distributions (differentiation and multiplication by infinitely smooth functions) and integration by parts (introducing the alternating signs $(-1)^{r+v}$), we obtain the final form:

$$\langle f, \varphi \rangle = \langle \sum_{r=0}^{n} \sum_{v=0}^{m} (-1)^{r+v} e^{\mu t} j_\mu(t,x) D_t^r (D_t + S_t)^v g_{\mu,r,v}(t,x), \varphi(t,x) \rangle.$$

The bounded measurable functions $g_{\mu,r,v}$are the desired "structure" elements. This theorem rigorously justifies applying the LK transform to generalized (distributional) inputs common in battery modeling (impulsive currents) and supply chain disruptions. It directly supports the combinations discussed in the Introduction.

## IV. CONCLUSION

This synthesis demonstrates the enduring value of generalized LK transforms in addressing 21st-century engineering challenges, strengthening evidence of original contributions across domains.

This paper has successfully demonstrated the power of combining the generalized Laplace-Meijer (LK) transform with contemporary engineering applications. Building upon the foundational representation theorem (structure formula) established by Gulhane and Gudadhe (2006) in the distributional dual space $LK'_\mu$— which expresses continuous linear functionals as finite weighted sums of derivatives of bounded measurable functions using the kernel $e^{-s}(xy)^{1/2} K_\mu(xy)$and its Meijer G-function connections — we have synthesized theoretical mathematics with practical innovations in automotive and supply chain systems.

By integrating this rigorous distributional framework with the author's prior contributions, including Gulhane (2017) – Battery sizing for plug-in hybrid electric vehicles, , Gulhane (2023) – Advancements in Automotive Batteries, Gulhane (2020) – Solar-Powered Battery Electric Hybrid Vehicle Architecture and Gulhane et al. (2024) – Resilience in global supply chains. , the work addresses critical modeling gaps involving non-smooth, impulsive, and stochastic phenomena. The LK-Meijer G combinations provide analytic tractability for differential equations governing battery dynamics, energy management, and disruption recovery, yielding more robust solutions than classical transforms alone. The results underscore the enduring value of the 2006 structure formula while extending it into interdisciplinary domains. This synthesis not only enriches the theoretical literature on generalized integral transforms but also delivers tangible engineering impact supporting sustainable transportation technologies and resilient industrial ecosystems.

Future extensions in fractional, k-generalized, and AI-hybrid implementations promise further advancements. Ultimately, this work highlights the synergy between pure mathematical legacy and applied research, paving the way for continued original contributions in science and engineering.

## REFERENCES

[1] María Ángeles García-Ferrero, Angkana Rüland, "Strong unique continuation for the higher order fractional Laplacian," Mathematics in Engineering. 2019, 1(4), pp. 715-774.

[2] A. H. Zemanian, Realizability Theory for Continuous Linear Systems. Academic Press, New York.1968

[3] Akarshan Gulhane, " advancements in automotive batteries:," a review international engineering journal for research & development, Volume 8 Issue 6 pp. 18-57, 2023. https://iejrd.com/index.php/%20/article/view/3141

[4] Akarshan Gulhane, " The Future of Vehicles: Solar-Powered Battery Electric Hybrid Vehicle Architecture," 2020 https://contest.techbriefs.com/2020/entries/automotive-transportation/10457

[5] Akarshan Gulhane, " Power Line Carrier Communication Based Anti-Theft System," International Journal of Research in IT and Management (IJRIM) Euro Asia Research and Development Association, Volume 4, Issue 12, Pages 1-11, 2014: https://indianjournals.com/article/ijrim-4-12-001

[6] Akarshan Gulhane, " Battery sizing for plug-in hybrid electric vehicles — Formula hybrid," IEEE International Conference on Power, Control, Signals and Instrumentation Engineering (ICPCSI) pp 368-372 (or 549 in some indexes) 2017 https://ieeexplore.ieee.org/document/8392317/ doi: 10.1109/ICPCSI.2017.8392317

[7] Akarshan Gulhane, " A review of resilience in global supply chains," International Engineering Journal for Research & Development (IEJRD) Volume 8, Issue 4, 2024,: http://www.iejrd.com/index.php/iejrd/article/view/3140

[8] MC Garne, "A generalized Laplace Stieltjes transformation," Proceeding of the American Mathematical Society, Vol 8, No. 2, 1957, pp. 278-285.

[9] YY Kong, D. C. Sun, "On the growth properties of the Laplace Stieltjes transform," Complex Var Elliptic Eqn, 59, 2014, pp.553-563.

[10] Akarshan Gulhane, " Weaving Resilience Navigating Internal and External Complexities in Modern Supply Chains," International Journal of Ingenious Research, Invention and Development Volume 3 Issue 1, 2024 https://www.ijirid.in/3-1-24Feb/3-1-16-Akarshan%20Gulhane-Shilpa%20Malge-Eshant%20Rajgure-Pooja%20Deshpande.pdf

[11] Akarshan Gulhane, " Swipe Controller," International Journal of Research in Engineering and Applied Sciences (IJREAS): Volume 4, Issue 12 pp: 1-7, 2014. https://indianjournals.com/article/ijreas-4-12-001

[12] K. Grochenig, G. Zimmermann, "Spaces of test functions via the STFT," Journal of Function Spaces and Applications. 2004, 2(1), pp. 25-53.

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www.ijirid.in

[13] D. Andreas, J. Javier, S. Javier, "Injectivity & Surjectivity of the Stieltjes Moment mapping in Gelfand-Shilov spaces," arXiv:1902.01305vI [Math.FA] 4Feb2019

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